**Stability Investigation and Stabilization of Nonlinear Switched Mechanical Systems via Decomposition**

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*A nonlinear hybrid mechanical system is studied. It is assumed that on the system homogeneous switched positional forces, linear gyroscopic forces and homogeneous dissipative ones act. Conditions are determined under which the trivial equilibrium position of the considered system is asymptotically stable for any admissible switching law. It is well known that to prove the asymptotic stability uniform with respect to switching law, it is sufficient to construct a common Lyapunov function for the family of subsystems corresponding to the hybrid system. However, till now there are no general constructive methods for finding of common Lyapunov functions, even for families composed of linear subsystems. The problem is especially difficult for mechanical systems with switched force fields, since such systems are described by differential equations of the second order. This results in the appearance of certain special properties of motions and essentially complicates the analysis of system dynamics. In particular, well known results obtained for switched systems of general form might be inefficient or even inapplicable for mechanical systems. In the present paper, a new approach to the stability analysis is proposed. The approach is based on the decomposition of the original system consisting of differential equations of the second order into two isolated first order subsystems of the same dimension. It should be noted that one of the isolated subsystem is switched, whereas another one is nonswitched. Thus, the decomposition method permits us to reduce the problem of a common Lyapunov function constructing for the original family of nonlinear systems of dimension with a special structure to that for an auxiliary family of subsystems of dimension which, generally, do not possess a special structure. The problem of stabilization of the equilibrium position of a system for any switching mode scaling the potential with the aid of small forces of radial correction is considered. For a model of the magnetic bearing of a rotor with nonlinear switched circular forces, the stabilizing feedback control law is constructed by the use of linear gyroscopic and nonlinear dissipative forces.*

***Keywords:*** *hybrid mechanical systems, asymptotic stability, control, Lyapunov functions, stabilization, decomposition, switching law*

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**References**

**1.** **Shorten R., Wirth F., Mason O., Wulf K., King C.** Stability Criteria for Switched and Hybrid Systems, *SIAM Rev.*, 2007, vol. 49, no. 4, pp. 545--592.

2. **Hai Lin, Antsaklis P. J.** Stability and Stabilizability of Switched Linear Systems: a Survey of Recent Results, *IEEE Trans. Automat. Contr.*, 2009, vol. 54, no. 2, pp. 308-322. DOI: [10.1109/TAC.2008.2012009](http://dx.doi.org/10.1109/TAC.2008.2012009)

**3. Liberzon D.** *Switching in Systems and Control*, Boston, MA, Birkhauser, 2003, 233 p.

**4. DeCarlo R., Branicky M., Pettersson S., Lennartson B.** Perspectives and Results on the Stability and Stabilisability of Hybrid Systems, *Proc. IEEE,* 2000, vol. 88, pp. 1069--1082.

5. **Vasil'ev S. N., Kosov A. A.** Analysis of dynamics of hybrid systems on the basis of common Lyapunov functions and multiple homomorphisms, *Avtomatika i Telemehanika*, 2011, no 6, pp. 27-47 (in Russian).

**6. Vassilyev S. N., Kosov A. A., Malikov A. I.** Stability Analysis of Nonlinear Switched Systems via Reduction Method, *Preprints of the 18th IFAC World Congress.* Milano, Italy. Aug. 28 – Sept. 2, 2011, pp. 5718-5723.

7. **Aleksandrov A. Ju., Kosov A. A.** On the stability and stabilization of equilibrium positions of nonlinear nonautonomous mechanical systems, *Izv. RAN. Teorija i Sistemy Upravlenija*, 2009, no. 4, pp. 13-23 (in Russian).

8. **Aleksandrov A. Ju., Kosov A. A.** On the stability and stabilization of nonlinear nonstationary mechanical systems, *Prikl. Matematika i Mehanika*, 2010, vol. 74, no. 5, pp. 774-788 (in Russian).

**9. Aleksandrov A. Ju., Kosov A. A., Chjen' Ja.** On the stability and stabilization of mechanical systems with switching, ***Avtomatika i Telemehanika*, 2011, no. 6, pp. 5--17** (in Russian).

10. **Matrosov V. M.** Method of Lyapunov Vector Functions: Analysis of Dynamical Properties of Nonlinear Systems. Moscow, Fizmatlit, 2001. 384 p. (in Russian).

 11. **Pjatnickij E. S.** Decomposition principle in dynamical systems control, *Dokl. AN SSSR*, 1988, vol. 300, no. 2, pp. 300-303 (in Russian).

12. **Chernous'ko F. L., Anan'evskij I. M., Reshmin S. A.** Methods of Nonlinear Mechanical Systems Control, Moscow, Fizmatlit, 2006. 328 p. (in Russian).

13. **Zubov V. I.** Analytical Dynamics of Gyroscopic Systems, Leningrad, Sudostroenie, 1970, 320 p. (in Russian).

14. **Merkin D. R.** Gyroscopic Systems, Moscow, Nauka, 1974. 344 p. (in Russian).

15. **Merkin D. R.** Introduction to the Theory of Motion Stability, Moscow, Nauka, 1987, 304 p. (in Russian).

16. **Zoteev V. E.** Parametric Identification of Dissipative Mechanical Systems on the Basis of Difference Equations, Moscow, Mashinostroenie, 2009, 344 p. (in Russian).

17. **Panovko G. Ja.** Foundations of Applied Theory of Oscillations and Impact, Leningrad, Mashinostroenie, 1976. 326 p. (in Russian).

18. **Gendelman O. V., Lamarque C. H.** Dynamics of linear oscillator coupled to strongly nonlinear attachment with multiple states of equilibrium, *Chaos, Solitons and Fractals,* 2005, vol.24, pp. 501–509.

19. **Gourdon E., Lamarque C. H.** Energy pumping for a larger span of energy,  *J. of Sound and Vibration,* 2005, vol. 285, pp. 711–720.

20. **Aleksandrov A. Ju., Kosov A. A., Platonov A. V., Fadeev S. S.** On the stability and stabilization of mechanical systems with switching of force fields, *Mehatronika, Avtomatizacija, Upravlenie*, 2013, no. 12, pp. 9–16 (in Russian).

21. **Aleksandrov A. Yu., Chen Y., Kosov A. A., Zhang L.** Stability of Hybrid Mechanical Systems with Switching Linear Force Fields, *Nonlinear Dynamics and Systems Theory*, 2011, vol. 11, no. 1, pp. 53--64.

22. **Zubov V. I.** Mathematical Methods of Automatic Control Systems Study, Leningrad, Sudostroenie, 1959. 324 p. (in Russian).

23. **Aleksandrov A. Yu., Kosov A. A., Platonov A. V.** On the Asymptotic Stability of Switched Homogeneous Systems, *Systems & Control Letters*, 2012, vol. 61, no. 1, pp. 127-133.

24. **Aleksandrov A. Ju., Zhabko A. P., Kosov A. A.** Analiz ustojchivosti i stabilizacija nelinejnyh sistem na osnove dekompozicii (Stability analysis and stabilization of nonlinear systems via decomposition), *Sib. mat. Zhurnal,* 2015 (in press) (in Russian).

25. **Post R. F.** Stability Issues in Ambient-Temperature Passive Magnetic Bearing Systems. February 17, 2000. *Lawrence Livermore National Laboratory. Technical Information Department’s Digital Library*, available at: <http://www.llnl.gov/tid/Library.html> .

26. **Metelicyn I. I.** K voprosu o giroskopicheskoj stabilizacii (On the gyroscopic stabilization), *Dokl. AN SSSR,* 1952, vol. 86, no. 1, pp. 31–34 (in Russian).